


Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year (تخلفات)		Final Term Exam Date: December 26, 2015 Course: Mathematics 1 – A Duration: 3 hours
<ul style="list-style-type: none"> The Exam consists of one page Answer All Questions 	<ul style="list-style-type: none"> No. of questions: 4 Total Mark: 100 	
<p>Question 1 Find y' from the following:</p> <p>(a) $y = x^4 + 2x^2 + 3x$ (b) $y = \cosh x^2 \cdot \sec 2x$ (c) $y = \cos x^2 - \ln \sin x$ (d) $y = \tan^{-1}x + \tan^{-3}x$ (e) $y^4 = x \log(x + y)$ (f) $y = t \sec t, x = t \sinh t$</p> <p>Question 2 (a) Find the following limits:</p> <p>(i) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^7 - 1}$ (ii) $\lim_{x \rightarrow 0} \frac{\ln(1 + 3x)}{2^x - 3^x}$ (iii) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3 + x^2}$ (iv) $\lim_{x \rightarrow \infty} \frac{x^8 + 2x}{x + x^9}$ (b) Write the Maclurin's series of the function: $f(x) = x \sin x$. (c) Show that : $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$. (d) Determine the extrema of : $f(x) = x^3 - 6x^2$ $g(x) = x^3 + 3$</p> <p>Question 3 Integrate the following:</p> <p>(a) $\int \frac{x^3}{(4 - x^2)^{1/2}} dx$ (b) $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$ (c) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ (d) $\int \tan^4 x dx$ (e) $\int x \sqrt{(x^2 + 1)} dx$ (f) $\int e^{3x} \sinh 2x dx$</p> <p>Question 4 (a) Find the area of the surface of revaluation generated by revolving about x-axis the cycloid $x = a(t - \sin t), y = (1 - \cos t), 0 \leq t \leq 2\pi$ (b) Find the area bounded by the curves: $y = x^3, x = 2, x = 5, y = 0$. (c) Find the volume generated by revolving, about x-axis, the area bounded by: $y = x^2, y = 0, x = 10$</p>		<p>24</p> <p>12</p> <p>4</p> <p>4</p> <p>6</p> <p>30</p> <p>7</p> <p>7</p> <p>6</p>

Good Luck

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Answer

Answer of Question 1

$$(a)(i) y' = 4x^3 + 2^{x^2} \cdot \ln 2 \cdot 2x + 3$$

$$(ii) y' = \cosh x^2 \cdot \sec 2x \cdot \tan 2x \cdot 2 + \sinh x^2 \cdot 2x \cdot \sec 2x$$

$$(iii) y' = -2x \sin x^2 - \frac{\cos x}{\sin x}$$

$$(iv) y' = \frac{1}{1+x^2} - 3 \tan^{-4} x \cdot \sec^2 x$$

$$(v) 4y^3 \cdot y' = 1 \cdot \log(x+y) + x \frac{1}{\ln 10} \cdot \frac{1+y'}{x+y}$$

$$\text{Then } y' \left(4y^3 - \frac{x}{\ln 10} \cdot \frac{1}{x+y} \right) = \log(x+y) + \frac{x}{\ln 10} \cdot \frac{1}{x+y}$$

$$\text{Then } y' = \frac{\log(x+y) + \frac{x}{\ln 10} \cdot \frac{1}{x+y}}{4y^3 - \frac{x}{\ln 10} \cdot \frac{1}{x+y}}$$

$$(vi) y' = \frac{\dot{y}}{\dot{x}} = \frac{t \sec t \cdot \tan t + \sec t}{t \cosh t + \sinh t}$$

-----24-Marks

Answer of Question 2

$$(a)(i) \lim_{x \rightarrow 0} \frac{\sqrt{x} - 1}{x^7 - 1} = \frac{0}{0} = \frac{1/2}{7} = \frac{1}{14}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{2^x - 3^x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\frac{\ln(1+3x)}{x}}{3^x \frac{((2/3)^x - 1)}{x}} = \lim_{x \rightarrow 0} \frac{1}{3^x} \frac{\frac{\ln(1+3x)}{x}}{\frac{((2/3)^x - 1)}{x}} = \frac{3}{\ln(2/3)}$$

$$(iii) \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3 + x^2} = \frac{0}{0}$$

Using L'Hopital's rule, we get

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3 + x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2 + 2x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\sin x}{6x + 2} = \frac{(0)}{0 + 2} = 0$$

$$(iv) \lim_{x \rightarrow \infty} \frac{x^8 + 2x}{x + x^9} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} + \frac{2}{x^8}}{\frac{1}{x^8} + 1} = \frac{0 + 0}{0 + 1} = 0$$

-----12-Marks

(b) Since $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$
 Then $f(x) = x \sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} \dots$

-----4-Marks

(c) Proof

-----4-Marks

(d) Since $f'(x) = 3x^2 - 12x = 3x(x - 4) = 0$

Then $x = 0$, $x = 4$.

Then, the point $(0, 0)$ is maximum because $f''(0) = 6x - 12 = -12$

the point $(4, -32)$ is minimum because $f''(4) = 6x - 12 = 12$

Since $g'(x) = 3x^2 = 0$

Then $x = 0$.

Using the first derivative test, we get

$$g'(0^-) = g'(-1) = 3x^2 = 3 = g'(1) = g'(0^+)$$

Then, the point $(0, 3)$ is neither maximum nor minimum.

-----4-Marks

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Answer of Question (3)

$$(a) \int \frac{x^3}{(4-x^2)^{1/2}} dx \text{ put } x = 2\sin\theta \quad \therefore dx = 2\cos\theta d\theta, \quad \cos\theta = \sqrt{4-x^2}$$

$$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta,$$

Substitute in the problem we have

$$\therefore \int \frac{x^3}{(4-x^2)^{1/2}} dx = \int \frac{8\sin^3\theta \cos\theta d\theta}{2\cos\theta} = 4\int \sin^3\theta d\theta = 4\int \sin^2\theta \cdot \sin\theta d\theta$$

$$= 4\int (1 - \cos^2\theta) \sin\theta d\theta = 4\int \sin\theta d\theta - 4\int \cos^2\theta \sin\theta d\theta$$

$$= -4\cos\theta + \frac{1}{3}\cos^3\theta + c$$

$$= -4\sqrt{4-x^2} + \frac{1}{3}(4-x^2)^{3/2} + c$$

$$(b) \int \frac{\sin x + \cos x}{\sin x - \cos x} dx$$

$$\int \frac{\sin x + \cos x}{\sin x - \cos x} dx = \int \frac{\sin x + \cos x}{\sin x - \cos x} \times \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int \frac{(\sin x + \cos x)^2}{\sin^2 x + \cos^2 x} dx$$

$$= \int (\sin x + \cos x)^2 dx = \int (\sin^2 x + 2\sin x \cos x + \cos^2 x) dx$$

$$= \int (1 + 2\sin x \cos x) dx = \int (1 + \sin 2x) dx = x - \frac{1}{2}\cos 2x + c$$

$$(c) \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \ln(e^x + e^{-x}) + C$$

$$\begin{aligned} (d) \int \tan^4 x dx &= \int \tan^2 x \tan^2 x dx = \int \tan^2 x (\sec^2 x - 1) dx \\ &= \int (\tan^2 x \sec^2 x - \tan^2 x) dx \\ &= \int (\tan^2 x \sec^2 x - (\sec^2 x - 1)) dx \\ &= \int (\tan^2 x \sec^2 x - \sec^2 x + 1) dx \\ &= \frac{1}{3} \tan^3 x - \tan x + x + c \end{aligned}$$

$$\begin{aligned} (e) \int x \sqrt{x^2 + 1} dx \quad \text{put } y = x^2 + 1 \text{ then } dy = 2x dx \\ \int x \sqrt{x^2 + 1} dx = \frac{1}{2} \int y^{1/2} dy = \frac{1}{2} \cdot \frac{2}{3} y^{3/2} + C = \frac{1}{3} (x^2 + 1)^{3/2} + C \end{aligned}$$

$$\begin{aligned} (f) \int e^{3x} \cosh 2x dx &= \int e^{3x} \frac{e^{2x} + e^{-2x}}{2} dx \\ &= \frac{1}{2} \int (e^{5x} + e^x) dx = \left(\frac{1}{10} e^{5x} + \frac{1}{2} e^x \right) + c \end{aligned}$$

-----30-Marks

Answer of Question (4)

$$(a) x = a(t - \sin t), y = a(1 - \cos t), 0 \leq t \leq 2\pi$$

$$x = a(t - \sin t), y = a(1 - \cos t) \quad \frac{dx}{dt} = a(1 - \cos t), \quad \frac{dy}{dt} = a \sin t$$

$$dL = \sqrt{\left[\frac{dx}{dt} \right]^2 + \left[\frac{dy}{dt} \right]^2} dt = \sqrt{a^2(1 - \cos t)^2 + a^2 \sin^2 t} = 2a \sin \frac{t}{2} dt$$

$$\begin{aligned}
\therefore S_x &= 2\pi \int_a^b y \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2} dt = 4\pi a \int_0^{2\pi} a(1 - \cos t) \sin \frac{t}{2} dt \\
&= 8\pi a^2 \int_0^{2\pi} \sin^3 \frac{t}{2} dt = 8\pi a^2 \int_0^{2\pi} \sin^2 \frac{t}{2} \sin \frac{t}{2} dt \\
&= 8\pi a^2 \int_0^{2\pi} (1 - \cos^2 \frac{t}{2}) \sin \frac{t}{2} dt \\
&= 8\pi a^2 \int_0^{2\pi} (\sin \frac{t}{2} - \cos^2 \frac{t}{2} \sin \frac{t}{2}) dt \\
&= -8\pi a^2 \left[2\cos \frac{t}{2} + \frac{2}{3} \cos^3 \frac{t}{2} \right]_0^{2\pi} = \\
&= -8\pi a^2 \left[\left(2\cos \frac{\pi}{2} + \frac{2}{3} \cos^3 \frac{\pi}{2} \right) - \left(2\cos 0 + \frac{2}{3} \cos^3 0 \right) \right] \\
&= -8\pi a^2 \left[0 - \left(2 + \frac{2}{3} \right) \right] = \frac{64}{3} a^2 \pi
\end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad A &= \int_2^5 y dx = \int_2^5 x^3 dx = \left[\frac{1}{4} x^4 \right]_2^5 = \frac{1}{4} [5^4 - 2^4] \\
&= \frac{625 - 16}{4} = \frac{609}{4} \text{ square unit}
\end{aligned}$$

$$\text{(c)} \quad V = \pi \int_{x=0}^{x=10} y^2 dx = \pi \int_0^{10} x^4 dx = \frac{\pi}{5} [x^5]_0^{10} = 20000\pi \text{ cubic unit}$$

-----20-Marks

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